NOTE ON THE THEORY OF IMAGES.

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Article 36, found upon pages 84–6, Vol. IV of this journal, is susceptible of the following interpretation:—

In that article nothing was said concerning the properties of i and j. It was implicitly assumed that, like ordinary algebraic quantities, they were commutative, associative, and distributive; and that $i^2 = j^2 = -1$.

In expanding $\varphi(x + iy + jz)$ we have

$$X + iY + jZ + ijW.$$

Now if we confine ourselves to space of three dimensions instead of four, we project the latter into the former by suppressing the term ijW; i. e. by *omitting all terms of the form* $Ki^m j^n y^m z^n$ *where* m *and* n *are both odd.*

By this interpretation what was called *the path of* U is really a projection of the same.

If $i, j, k (\equiv ij)$ be commutative, associative, and distributive, then generally $\varphi(x + iy + jz + kw)$ is developable in powers of $iy + jz + kw$ and is

$$X + iY + jZ + kW;$$

and in four-dimentional space the locus of the point $xyzw$ has an image, the locus of $XYZW$. Now if we suppress kW we project the path of U, the image of the path of $u (= x + iy + jz + 0)$, into the $(1, i, j)$-space.

Professor Oliver suggests that by writing $\alpha \equiv 1$, $\beta \equiv i\sqrt{-1}$, $\gamma \equiv j\sqrt{-1}$, $\delta \equiv -k$, this $(1, i, j, k)$-algebra takes the symmetric form

$$\begin{aligned}
\alpha^2 &= \beta^2 = \gamma^2 = \delta^2, \\
\alpha\beta &= \beta\alpha = \gamma\delta = \delta\gamma, \\
\alpha\gamma &= \gamma\alpha = \delta\beta = \beta\delta, \\
\alpha\delta &= \delta\alpha = \beta\gamma = \gamma\beta.
\end{aligned}$$